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JB-003-1016002 Seat No. _____

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2016) Examination

August - 2019

Mathematics - Paper - 9

(Mathematical Analysis - II & Abstract Algebra - II) (Theory)

Faculty Code : 003

Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 (a) Answer briefly the following questions : **4**

(1) Define Compact set.

(2) Define similar set.

(3) Find the Greatest lower bound of $\left\{\frac{1}{n} / n \in N\right\}$.

(4) Show that $R - \{2\}$ is connected.

(b) Answer any **one** out of two : **2**

(1) Show that $A = (1, 3)$ and $B = (3, 5)$ are separated set of metric space R.

(2) For the set $E = \left[\frac{1}{3}, \frac{2}{3}\right]$ the collection $\{G_n / n \in N\}$ where $G_n = \left(\frac{1}{n}, 1\right)$ is a cover of E or not?

(c) Answer any **one** out of two : **3**

(1) Determine the subset $\{-1\} \cup (0, \infty)$ of metric space R are Open, Closed, Compact and Connected.

(2) If (X, d) is a compact metric space, Then X is totally bounded set.

(d) Answer any **one** out of two : **5**

(1) A metric space (X, d) is sequential compact iff it satisfies Bolzano Weirstrass theorem.

(2) Prove that set of all real numbers R is not Countable.

2 (a) Answer the following questions briefly : **4**

(1) Find $L^{-1}\left(\frac{s}{s^2+9}\right)$

(2) Find $L^{-1}\left(\frac{s-1}{(s-1)^2-9}\right)$

(3) Find $L\left(e^{2t}\sin 3t\right)$.

(4) Find $L^{-1}\left(t^{-1/2}\right)$.

(b) Answer any **one** out of two : **2**

(1) Find Laplace transformation of $f(t)=\begin{cases} e^t, t \le 2 \\ 3, t > 2 \end{cases}$.

(2) State and prove First Shifting theorem.

(c) Answer any **one** out of two : **3**

(1) Find Inverse Laplace Transformation of $\frac{3s+4}{s^2+16}$.

(2) Find Inverse Laplace Transformation of $\frac{s+1}{s^2+s+1}$.

(d) Answer any **one** out of two : **5**

(1) Find Inverse Laplace Transformation of $\frac{4s+5}{(s-1)^2+(s+2)}$.

(2) Find Inverse Laplace Transformation of $\frac{1}{\left(s^2+2s+5\right)^2}$.

3 (a) Answer the following questions briefly : **4**

(1) Find $L\left(t\sinh t\right)$.

(2) Find $L\left(t^3e^{-3t}\right)$.

(3) Find $L\left(e^t\cosh 2t\right)$.

(4) Find $L^{-1}\left(\frac{1}{\left(s^2+a^2\right)^2}\right)$.

(b) Answer any **one** out of two : **2**

(1) Find $L\left(te^{2t}\cos 3t\right)$.

(2) Find $L\left(\frac{1-e^{t}}{t}\right)$.

(c) Answer any **one** out of two : **3**

(1) Find $L^{-1}\left(\frac{1}{s\left(s^2+a^2\right)}\right)$

(2) Find $y''-y=t, y(0)=y'(0)=1$.

(d) Answer any **one** out of two : **5**

(1) Using Convolution theorem find

$$L^{-1}\left(\frac{1}{(s+1)\left(s^2-2s+2\right)}\right)$$

(2) Solve $y''+ky'-2k^2\,y=0, y'(0)=2k$.

4 (a) Answer the following questions briefly : **4**

(1) Obtain radicals of the rings $\left(Z_{12},+_{12},\times_{12}\right)$.

(2) Define Natural Mapping.

(3) Define Skew - field.

(4) Define Sub integral domain.

(b) Answer any **one** out of two : **2**

(1) $U_1=\left\{f\in C[0,1]/\,f(0)=0\right\}$ is subring of $\left(C[0,1],+,*\right)$.

(2) Let $\phi:(G,*)\to(G',\Delta)$ be a homomorphism then If N is normal subgroup of G then, $\phi(N)$ is a normal Subgroup of $\phi(G)$.

(c) Answer any **one** out of two : **3**

(1) Prove that a field has no proper Ideal

(2) A homomorphism $\phi:(G,*)\to(G',\Delta)$ is one - one iff $K_\phi=\{e\}$.

(d) Answer any **one** out of two : **5**

(1) State and prove First Fundamental Theorem of Homomorphism

(2) A commutative ring with unity is a field if it has no proper Ideal.

5 (a) Answer briefly the following questions : **4**

(1) Find conjugate of $a = -1 + 2i - 3j + k$.

(2) Find norm of $2 + i + 2j + 4k$.

(3) What do you mean by Quadratic polynomials ?

(4) What do you mean by Linear Polynomials ?

(b) Answer any **one** out of two : **2**

(1) Simplify : $(1 + 2j - 3k)^{-2}$.

(2) If $f = (2, 0, -3, 0, 4, 0.....)$ and $g = (1, -2, 0, 0.....)$ are polynomials of $R[X]$ then find $f + g$.

(c) Answer any **one** out of two : **3**

(1) State and prove Remainder theorem

(2) State and prove Factor theorem

(d) Answer any **one** out of two : **5**

(1) Factorize $f(x) = x^4 + 4 \in Z_5[X]$ by using Factor theorem.

(2) Find g. c. d. of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and $g(x) = 2x^3 - 3x + 5 \in R[X]$ and express it in the form $a(x) f(x) + b(x) g(x)$.